INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.15-2.16]

POWER AND EXPONENTIAL MODELS

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 8]

(a) Describe the relation between y and x in each equation below: [4]

$y = 0.2x$	
$y = 3.7x^2$	
$y = \frac{2.15}{x}$	
$y = \frac{1}{3x^2}$	

(b Given that k is a constant, describe the relation between P and Q in each relation below: [4]

$P = kQ$	
$P = kQ^3$	
$P^2 = kQ^3$	
$PQ^2 = k$	

2. [Maximum mark: 4]

(a) Consider the free fall formula $s = \frac{1}{2}gt^2$, where

s : distance travelled, t : mass, g : constant acceleration due to gravity.

Describe the relation between distant travelled and time.

[1]

(b) Consider the relation $F = mg$, where

F : force, m : mass, g : acceleration due to gravity.

Describe the relation between any two variables when the third variable remains constant.

If g remains constant	
If m remains constant	
If F remains constant	

[3]

3. [Maximum mark: 4]

The variable S is **inversely proportional** to the square of the variable T.

Given that $S = 10$ when $T = 0.2$, find a formula that expresses S in terms of T.

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4. [Maximum mark: 8]

The power function model $P = ax^n$ has **two** parameters to be determined, a and n.

The following table of data is given

x	1	2
P	10	80

(a) Find the values of the parameters a and n and write down the final form of the model. [3]

(b) Find the value of P when $x = 1.5$, [1]

(c) Find the **exact** value of x when $P = 20$. [2]

(d) Sketch the graph of the model; Indicate the data of the table above. [2]

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5. [Maximum mark: 8]

The power function model $P = aQ^n$, $Q \geq 0$, has **two** parameters to be determined, a and n.

The following table of data is given

Q	2	4
P	1.2	4.8

(a) Find the value of (i) n, (ii) a. [4]

(b) Find the value of Q when $P = 2.7$, [2]

(c) Sketch the graph of the model; Indicate the data of the table above. [2]

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6. [Maximum mark: 8]

The power function model $P = aQ^{-n} = \dfrac{a}{Q^n}$, $Q > 0$, has **two** parameters to be determined, a and n.

The following table of data is given

Q	2	4
P	8	1

(a) Find the value of (i) n, (ii) a. [3]

(b) Write down the final form of the model and sketch its graph; Indicate the data of the table above. [3]

(c) Find

(i) The value of Q when $P = 0.001$,

(ii) The least integer Q, such that $P < 0.001$. [2]

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7. [Maximum mark: 8]

The exponential model $P = Ae^{kt}$ has **two** parameters to be determined, A and k.

The following table of data is given

t	0	2
P	10	20

(a) Write down the value of the parameter A. [1]

(b) Show that $k = \frac{\ln 2}{2}$ and express k it in 3 significant figures. [3]

(c) Write down the final form of the model and sketch its graph; Indicate the data of the table above. [3]

(d) Use regression on your GDC to find the best fit exponential model. [1]

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8. [Maximum mark: 9]

The exponential model $P = P_0 e^{kt}$ has **two** parameters to be determined, P_0 and k .

The following table of data is given

t	1	2
P	10	20

(a) Find the parameters P_0 and k **analytically**. [4]

(b) Write down the final form of the model and sketch its graph; Indicate the data of the table above. [3]

(c) The value of P exceeds 30 when $t = t_1$. Find the value of t_1. [1]

(d) Use regression on your GDC to find the best fit exponential model. [1]

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9. [Maximum mark: 10] [mainly for HL]

For the exponential model $P = P_0 e^{kt} + c$ the following table of data is given

t	1	2	3
P	20	30	50

(a) Write down **three** relations in the P_0, k and c (the parameters of the model). [2]

(b) Use the first two relations to obtain a new relation between P_0 and k only. [2]

(c) Use the last two relations to obtain another relation between P_0 and k only. [1]

(d) **Hence** find the parameters P_0 and k, and then the parameter c. [4]

(e) Write down the final form of the model. [1]

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10. [Maximum mark: 10] [mainly for HL]

For the exponential model $P = P_0 e^{kt} + c$ the following table of data is given

t	1	2	3
P	75.50	63.63	53.90

(a) Write down **three** relations in P_0, k and c. [2]

(b) Show that $k = -0.2$, correct to 1dp. [4]

(c) Find the parameters P_0 and c. [3]

(d) Write down the final form of the model. [1]

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11. [Maximum mark: 7]

A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10\,e^{0.4t}$.

(a) Find the initial number of leopards [1]

(b) How many leopards are there after 2 years? [2]

(c) How long will it take for the number of leopards to reach 100? [2]

(d) How long will it take for the number of leopards to double? [2]

Give your answers to an appropriate degree of accuracy.

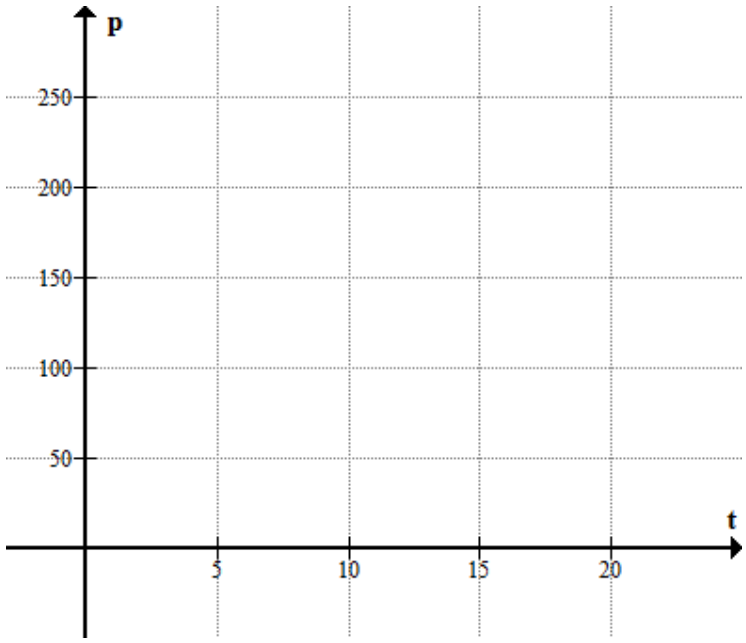
12. [Maximum mark: 7]

The population p of bacteria at time t is given by $p = 100e^{0.05t}$. Calculate

(a) the value of p when $t = 0$; [2]

(b) How long will it take for the size of the population to double? [2]

(c) Sketch the graph of the population of bacteria (p) in terms of time (t) on the diagram below; Show the information found in parts (a) and (b). [3]



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13. [Maximum mark: 8]

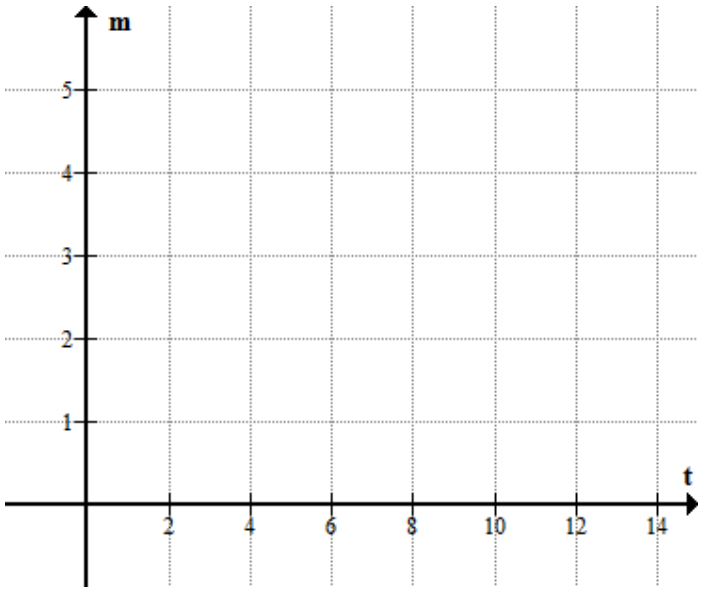
The mass m kg of a radio-active substance at time t hours is given by $m = 4e^{-0.2t}$.

(a) Write down the initial mass. [1]

(b) The mass is reduced to 1.5 kg. How long does this take? [2]

(c) Find the half-life time of m. [2]

(d) Sketch a graph of mass m in terms of time t (in hours) on the diagram below; Show the information found in parts (a), (b) and (c). [3]



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14 [Maximum mark: 6]

A machine was purchased for \$10000. Its value V after t years is given by $V = 10000e^{-0.3t}$. The machine must be replaced at the end of the year in which its value drops below \$1500.

(a) Determine in how many years the machine will need to be replaced. [3]

(b) Sketch a graph of Value V in terms of time t (in years); Show the information found above. *(Give an appropriate scale for each axis)*. [3]

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15. [Maximum mark: 8]

The area A km^2 affected by a forest fire at time t hours is given by $A = A_0\,\mathrm{e}^{kt}$.

When $t = 5$, the area affected is 1 km^2. Given that $A_0 = \frac{1}{\mathrm{e}}$,

(a) Show that $k = 0.2$; [3]

(b) Find the value of t when 100 km^2 are affected. [3]

(c) Sketch a graph of area A in terms of time t (in hours); Show the information found in part (b) above. *(Give an appropriate scale for each axis)* [2]

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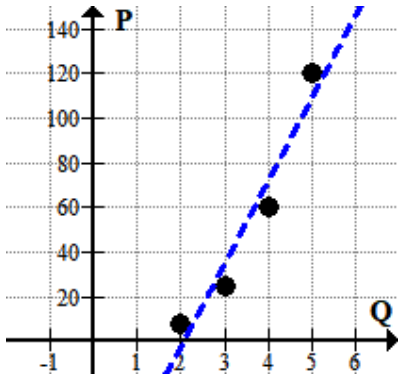
16. [Maximum mark: 7]

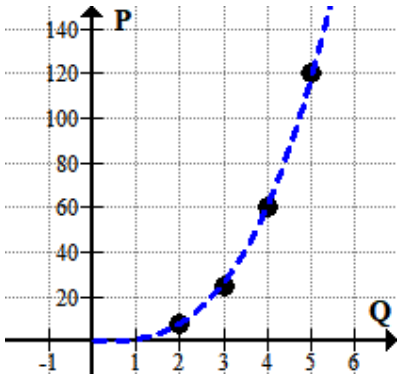
P is an increasing function of Q, where $Q > 0$. An experiment gave the following data.

Q	2	3	4	5
P	8	25	60	120

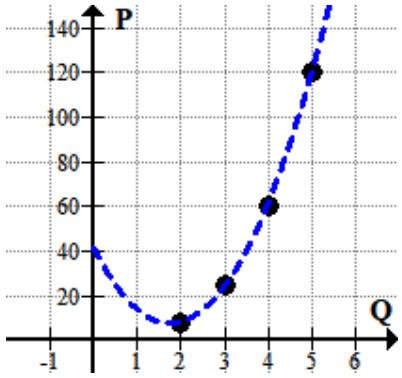
For models are suggested for this point series.



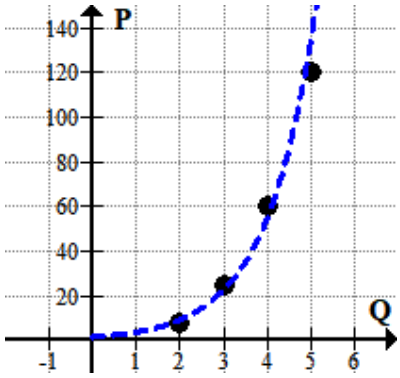
LINEAR



POWER



QUADRATIC



EXPONENTIAL

(a) Use regression on your GDC to write down the formulas of the four models. [5]

LINEAR:	**POWER:**
QUADRATIC:	
EXPONENTIAL: (ae^{bx}):	($a \cdot b^x$):

(b) Which model do you suggest for the given point series? [2]

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B. Paper 2 questions (LONG)

17. [Maximum mark: 15]

There were 1420 doctors working in a city on 1 January 1994. After n years the number of doctors, D, working in the city is given by

$$D = 1420 + 100n.$$

(a) (i) How many doctors were there working in the city at the start of 2004?

(ii) In what year were there first more than 2000 doctors working in the city? [3]

At the beginning of 1994 the city had a population of 1.2 million. After n years, the population, P, of the city is given by

$$P = 1\,200\,000\,(1.025)^n.$$

(b) (i) Find the population P at the beginning of 2004.

(ii) Calculate the percentage growth in population between 1 January 1994 and 1 January 2004.

(iii) In what year will the population first become greater than 2 million? [7]

(c) (i) What was the average number of people per doctor at the beginning of 1994?

(ii) After how many **complete** years will the number of people per doctor first fall below 600? [5]

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18. [Maximum mark: 10]

Initially a tank contains 10 000 litres of liquid. At the time $t = 0$ minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, V litres, which remains in the tank after t minutes is given by

$$V = 10\,000\,(0.933^t).$$

(a) Find the value of V after 5 minutes. [1]

(b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank. [3]

(c) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen. [3]

(d) (i) Find the value of $10\,000 - V$ when $t = 0.001$ minutes.

(ii) Hence or otherwise, estimate the initial flow rate of the liquid.
Give your answer in litres per minute, correct to two significant figures. [3]

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